Assignment 3: OLS, Binary Logit, BLP and Petrin & Train control function method

Yi Zhu and Abhishek Borah

1. Model Summary

1.1 Binary Logit Model (BL)

Demand side: Consumer *i* either chooses or does not choose product *j* at a time in market *t* (*t*=1,…,*T*)

The utility of consumer *i* choosing product j=1 is given by:

$u_{i1t} = \beta - \alpha p_{1t} + \xi_{1t} + \varepsilon_{i1t}$, and that of the outside option is $u_{i0t} = \varepsilon_{i0t}$.

Assuming the error terms ε_{ijt} where (j=0, 1) are distributed i.i.d. Type I extreme value, the market-share for *j=1* is then given by:

$$s_{1t}(p, x, \xi; \theta) = \frac{e^{\beta - \alpha p_{1t} + \xi_{1t}}}{(1 + e^{\beta - \alpha p_{1t} + \xi_{1t}})}, \quad (1)$$

Define the mean utility as $\delta_{1t} = \beta - \alpha p_{1t} + \xi_{1t}$, the above equation implies that

$$\delta_{1t} = \ln(s_{1t}) - \ln(s_{0t}) = \beta - \alpha p_{1t} + \xi_{1t}. \quad (2)$$

Supply side: We assume the firm's marginal cost function is constant and linear in cost shifters (w_{1t}), i.e.

$$\log mc_{1t} = w_{1t}\gamma + \omega_{1t}, \quad (3)$$

Given the above demand system in equation (1), the profits of firm *j* are

$$\Pi_{1t} = (p_{1t} - mc_{1t}) M s_{1t}(p, \xi; \theta),$$

where *M* is market size.

Assuming the existence of a pure-strategy interior equilibrium, the price satisfies the first order condition:

$$p_{1t} = \exp(w_{1t}\gamma + \omega_{1t}) + \frac{1}{\alpha}\frac{1}{1 - s_{1t}} \quad (4)$$

Parameters that need to be estimated are $\alpha, \beta,$ and γ.

1.2 The Random Coefficient Logit model (RCL)

Demand Side: Considering heterogeneity in consumers' taste preferences, the utility of consumer *i* choosing product j=1 is given by:

$$u_{i1t} = -\alpha p_{1t} + \overline{\beta} + \xi_{jt} + \sigma v_i + \varepsilon_{i1t}, \tag{5}$$

and the utility of choosing outside option is $u_{i0t} = \varepsilon_{i0t}$,

v_i is an unobserved consumer characteristic with standard normal distribution, i.e. $v_i \sim N(0,1)$, and ε_{ijt} (where j =0,1) is consumer's idiosyncratic taste preferences and assumed to be distributed i.i.d. Type I extreme value.

Defining the set of consumer unobservables that lead to the consumption of product j=1 as

$$A_{1t} = \{v_i, \varepsilon_{i\cdot t} \mid u_{i1t} > u_{i0t}\}, \tag{6}$$

Then the market share of product j is given by:

$$s_{1t}(p, \xi; \alpha, \overline{\beta}, \sigma) = \int_{A_{1t}} \frac{e^{\delta_{1t} + \sigma v_i}}{(1 + e^{\delta_{1t} + \sigma v_i})} dF(v) \tag{7}$$

where the mean utility is $\delta_{1t} = -\alpha p_{1t} + \overline{\beta} + \xi_{1t}$ and $F(v)$ is the distribution of vector v.

Supply side:

Similar to what we have in the Binary Logit setting, the firms sets its prices to maximize profits:

$$(p_{1t} - mc_{1t}) M s_{1t}(p, \xi; \alpha, \overline{\beta}, \sigma).$$

The first order condition is:

$$s_{1t}(p, \xi; \alpha, \overline{\beta}, \sigma) + (p_{1t} - mc_{1t}) \int_{v_i} \frac{e^{\delta_{1t} + \sigma v_i}}{(1 + e^{\delta_{1t} + \sigma v_i})^2} (-\alpha) dF(v) = 0 \tag{8}$$

Again, if we assume constant marginal cost function as equation (3), re-arranging the FOC in equation (8) in vector notation, we can get

$$log\{p - \Delta^{-1} s(p, \xi; \alpha, \overline{\beta}, \sigma)\} = w\gamma + \omega, \tag{9}$$

where $\Delta = \dfrac{\partial s(p, \xi; \alpha, \overline{\beta}, \sigma)}{\partial p} = \displaystyle\int_{v_i} \frac{e^{\delta_{1t} + \sigma v_i}}{(1 + e^{\delta_{1t} + \sigma v_i})^2} (-\alpha) dF(v)$.

Therefore, the parameters that you need to estimate are $\alpha, \overline{\beta}, \sigma,$ and, γ .

1.3 Petrin & Train (2005)

Demand function for product j=1 is given as

$$u_{i1t} = -\alpha p_{1t} + \beta + \xi_{jt} + \varepsilon_{i1t} \quad , \tag{10}$$

Prices are a function of cost-shifters w and error terms ξ . We write the reduced form for prices as:

$$P = p(w, \xi) \tag{11}$$

By OLS regression, we can get

$$E(P_1 \mid w) = \hat{P}_{1t} = \gamma_0 + \gamma_1 w_{1t} \tag{12}$$

$$g_1(\xi_t) = P_{1t} - \hat{P}_{1t} \tag{13}$$

Then the control function is given as

$$\varepsilon_{i1t} = \lambda g_1(\xi_t) + \zeta_{i1t} \tag{14}$$

We can have alternative specifications for this control functions as described in Petrin and Train (2005), which needs more parameters to be estimated.

We could also have a non-parametric specification where for period t =1, 2…T, one estimator for the value of the control for market t*,

$$\hat{\tilde{\xi}}_{t^*} = \frac{\sum_{t=1}^{T} 1(p_t \le p_{t^*}) * 1(w_t = w_{t^*})}{\sum_{t=1}^{T} 1(w_t = w_{t^*})} \tag{15}$$

In this non parametric case, $g_1(\xi_t) = \hat{\tilde{\xi}}_{t^*}$

Incorporating this calculated value in equation 14, the demand function changes into:

$$u_{i1t} = -\alpha p_{1t} + \beta + \lambda g_1(\xi_t) + \zeta_{i1t} \tag{16}$$

where ζ_{i1t} are distributed i.i.d. Type I extreme value

As a next step, calculate the market shares and find parameters where

$$\hat{\alpha}, \hat{\beta}, \hat{\lambda} = \underset{\alpha,\beta,\lambda}{\arg\min} \sum_{t=1}^{t=T} \left| s_{1t} - \left(\frac{e^{-\alpha p_{1t} + \beta + \lambda \hat{\tilde{\xi}}_t}}{1 + e^{-\alpha p_{1t} + \beta + \lambda \hat{\tilde{\xi}}_t}} \right) \right|$$ where s_{1t} is the observed market share

2. Estimation Exercise

The estimation exercise has four parts and you need to use two datasets. The estimations are

1) Ordinary least squares regression (OLS)

2) Binary Logit (BL)

3) Random Coefficients Logit (RCL)

4) Petrin and Train method

Thereafter, you need to compare your estimates of OLS with BL and explain which estimates are better and why?

2.1 Data Description

- The data has market share information for one product (*J*=1) in 500 markets (*T*=500).
- There is price (*p*) information for the demand side of the model and one cost shifter (*w*) which affects the marginal costs in the supply side.
- For OLS, BL and Petrin & Train, use **file name: data_mnl** and for RCL and use **file name: data_rcl.** In both files, the first column is market share, the second is price, and the third column is w.

2.2 Estimations

OLS - Use ordinary least square regression to estimate demand (equation 2) and recover the parameters α, β .

BL – Use Generalized Method of Moments (GMM) for joint estimation of demand (equation 2) and supply (equation 4) and recover the parameters $\alpha, \beta,$ and γ

You would need to use **moment conditions** here:

E$\begin{bmatrix} z'\xi \\ z'\omega \end{bmatrix}$=0 where ξ and ω are vectors of error terms of the demand and supply side respectively.

Denoting $\begin{bmatrix} z'\xi \\ z'\omega \end{bmatrix}$ as m, the objective function you need to minimize is: m'*Inv(W)*m where W is the weight matrix. Setting W equal to an identity matrix will recover the right parameters in this application.

z is a set of instrument variables such as constant and w (cost shifter). Do also give a reason why price cannot be an instrument here.

RCL – Use GMM to recover $\alpha, \bar{\beta}, \sigma,$ and, γ, using BLP approach. Use moment conditions and find appropriate instruments for the same. As mentioned in class, you would have to employ the contraction mapping approach. You could refer Nevo's code mentioned below (m-file meanval.m) for getting a notion about the contraction mapping approach.

Petrin and Train –First stage, use the non parametric approach to calculate the $\hat{\tilde{\xi}}$. Instead of $w_t = w_{t*}$ (you would obtain a vector of ones if you do as the w's are continuous) in equation 15 use a different tolerance level, $|w_t - w_{t*}| < tol$ where *tol* could range from 0 to .5 to calculate the $\hat{\tilde{\xi}}$'s. Use the specification given in equation 14 where ζ_{i1t} are distributed i.i.d. Type I extreme value.

In the next stage, minimize the distance between observed and calculated market shares as explained in 1.3. The minimization would enable you to recover the demand parameters α and β. Also see how the tolerance level affects the estimates.

2.3 Estimation Tips

- The parameters are all positive integers. It seems that starting parameter values do not matter.
- For the RCL case, use simulation to calculate the derivative of market share with respect to price.
- Avoid using 'for' loop! Vectorize your code, especially for the contraction mapping program!
- It will take a couple of hours for the RCL case to converge. Try to run on a fast computer. Given below is a link to a repository of BLP codes originating from Nevo.

  http://rasmusen.org/g604/lectures/blp/frontpage.htm

  Also attached is a link to a paper by Rasmusen which you may find helpful.

  http://www.bus.indiana.edu/riharbau/RePEc/iuk/wpaper/bepp2006-04-rasmusen.pdf

- When you use simulation, remember to hold the draws constant over the optimization!

---------------------------------------All the best---